MVT, anti-derivatives and integration

November 18, 2016

Problems

Problem 1. Two horses start a race at the same time and finish in a tie. Prove that at some time during the race they have the same speed. (Hint: MVT.)

Problem 2. Let $f(x) = \frac{1}{x^2}$, and $F(x)$ be an antiderivative of f with the property $F(1) = 1$. **True or false:** $F(-1) = 3$.

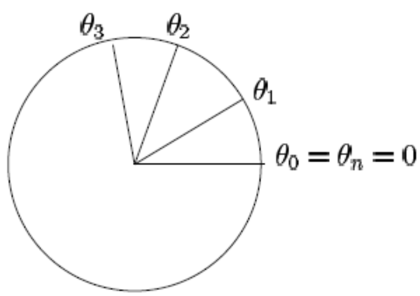
Problem 3. A rocket lifts off the surface of Earth with a constant acceleration of 20 m/sec^2. How fast will the rocket be going 1 minute later?

Problem 4. Compute the sum $\sum_{i=3}^{n} (i-2)^2$. You can use the formulas we've seen in class.

Problem 5. Compute $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{i(i+1)}$. (Hint: $\frac{1}{2\cdot 3} = \frac{1}{2} - \frac{1}{3}$)

Problem 6. Compute the integral $\int_0^2 x dx$ **by definition**. Verify that this answer is the same as the usual (geometric) area under the graph of $f(x) = x$ over $[0, 2]$.

Problem 7. We cut a circular disk of radius r into n circular sectors, as shown in the figure, by marking the angles θ_i at which we make the cuts ($\theta_0 = \theta_n$ can be considered to be angle 0). A circular sector between two angles θ_i and θ_{i+1} has area $\frac{1}{2}r^2\Delta\theta$, where $\Delta\theta = \theta_{i+1} - \theta_i$.



We let $A_n = \sum_{i=0}^{n-1} \frac{1}{2}r^2\Delta\theta_i$. Then the area of the disk, A, is given by

1. A_n, independent of how many sectors we cut the disk into.
2. $\lim_{n\to\infty} A_n$.
3. $\int_0^{2\pi} \frac{1}{2}r^2 d\theta$.
4. all of the above.